JHEP HYPER VERSION

Received: *August 31, 1999*, Accepted: *October 26, 1999*

Spin-two glueballs, positive energy theorems and the AdS/CFT correspondence

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Abstract: We determine the spectrum of graviton excitations in the background geometry of the AdS soliton in $p+2$ dimensions. Via the AdS/CFT correspondence, this corresponds to determining the spectrum of spin-two excitations in the dual effective p-dimensional field theories. For the cases of D3- and M5-branes, these are the spin-two glueballs of QCD_3 and QCD_4, respectively. For all values of p, we find an exact degeneracy of the spectra of these tensor states and certain scalar excitations. Our results also extend the perturbative proof of a positive energy conjecture for asymptotically locally AdS spacetimes (originally proposed for p=3) to an arbitrary number of dimensions.

Keywords: Strong Coupling Expansion, D-branes, Conformal Field Models in String Theory, Classical Theories of Gravity.

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Contents

1. Introduction

The AdS/CFT correspondence [1, 2, 3] — for a comprehensive review, see ref. [4] — has provided new perspectives on the holographic principle [5], which asserts that a consistent theory of quantum gravity in d dimensions must have an alternate formulation in terms of a nongravitational theory in $d-1$ dimensions. This equivalence is implemented in the AdS/CFT correspondence with a duality between a gravitational theory in d-dimensional anti-de Sitter space and a conformal field theory on a $(d-1)$-dimensional "boundary" space. Using this correspondence, one can gain new insights into both of the theories on either side of the duality. On the one hand, quantum gravity is reformulated as an ordinary quantum field theory and so one has a new framework with which to study the perplexing puzzles surrounding black holes and Hawking evaporation. On the other hand, the duality has become a useful tool with which to study a wide class of strongly coupled field theories in a non-perturbative framework.

In the latter context, the AdS/CFT correspondence has been extensively used to study "QCD-like" field theories by considering a variety of solutions to AdS supergravity — see, for example, refs. [6]–[12]. The original proposal by Witten [6] was made in the setting of string theory, where the conformal field theory is a supersymmetric gauge theory. One can describe ordinary (i.e. nonsupersymmetric) Yang-Mills

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theory by compactifying one of the spatial directions on a "small" circle and imposing antiperiodic boundary conditions on the fermions around this direction. In this case, the additional fermions and scalars appearing in the supersymmetric field theory would acquire large masses of the order of the compactification scale, leaving the gauge fields as the only low energy degrees of freedom. On the supergravity side, this proposal corresponds to considering spacetimes which are asymptotically AdS locally, but not globally. The relevant supergravity solution was found [6] to be the "AdS soliton" — following the nomenclature of ref. [13] — that is the double analytic continuation of a planar AdS black hole. In this geometry, the circle direction smoothly contracts to a point in the interior and as a result of this nontrivial topology, the supergravity fermions (and their dual gauge theory counterparts) are antiperiodic on the asymptotic circle.

In the special case of supergravity on AdS_5, which arises in the throat geometry of D3-branes, the dual field theory is four-dimensional $\mathcal{N} = 4$ SU(N) super-Yang-Mills theory and so upon compactification, the above construction produces an effective model of three-dimensional ordinary Yang-Mills theory, i.e. QCD_3 [6]. Similarly beginning with AdS_7, which arises from M5-branes, and compactifying two directions, one produces a model of QCD_4 [6]. This idea has been exploited [3, 7] to make predictions about the large-N behavior of these QCD theories at strong coupling. Specifically, it was shown [6] that these theories generically produce an area law for spatial Wilson Loops and possess a mass gap, both of which are evidence for confinement. The spectrum of scalar glueballs for $QCD_{3,4}$ has been calculated [7], and found to be remarkably similar to that calculated by lattice techniques [14, 15, 16]. A similar scalar spectrum was calculated for arbitrary Dp-branes in ref. [10].

In this paper we will extend the work of refs. [7, 10] by calculating the complete spectrum of spin-two excitations of the effectively p-dimensional field theories which are dual to the AdS soliton in $p+2$ dimensions. In the cases, for which $p = 3$ or $p = 5$, this amounts to a determination of the spin-two glueball spectra in QCD_3 and QCD_4. For all values of p, we find a surprising degeneracy between the spin-two states and scalar excitations associated with a minimally coupled massless scalar field propagating on the AdS soliton background.

Another interesting development motivated by the AdS/CFT correspondence was the proposal of a new positive energy conjecture for asymptotically locally AdS spaces [13]. This investigation was originally motivated by the observation that in asymptotically flat spacetimes geometries in which an asymptotic circle is contractible are unstable because the energy can be arbitrarily negative [17]. For gravity with a negative cosmological constant, one does find that the AdS soliton has a finite negative energy. However, in ref. [13] for five dimensions, it was shown that this solution is perturbatively stable and that the negative energy is naturally identified with the Casimir energy of the dual field theory — see also ref. [18]. Hence motivated by the AdS/CFT correspondence, it was proposed that the AdS soliton is in fact the minimum energy solution with the given asymptotic structure. Our present calculation extend the results of ref. [13] by providing a perturbative proof of the stability of the AdS soliton which holds in all dimensions $d \geq 4$. Hence we are lead to extend the positive energy conjecture of ref. [13] beyond five dimensions, which was the focus of their discussion.

The paper is organized as follows: In section 2, we review the standard approach to calculating glueball spectra using AdS/CFT correspondence. In section 3, we calculate the spectrum of spin-two excitations in the field theory dual to the AdS soliton in arbitrary dimensions. In section 4, we consider our results as a perturbative proof of the positive energy conjecture of ref. [13] extended to arbitrary dimensions. section 5 contains a further discussion of our results.

2. Review of scalar spectra

In the framework of string theory where the AdS/CFT duality is best understood, simple backgrounds are both supersymmetric and conformally invariant. Both of these symmetries must be eliminated to construct models of real world QCD. Witten's suggestion [6] described above breaks supersymmetry with the antiperiodic fermion boundary conditions, and breaks the conformal invariance by introducing a scale, namely the size of the circle. The appropriate supergravity background satisfying the desired boundary conditions is the AdS soliton. For this solution, Witten argued that the supergravity fields must have a discrete spectrum, and hence a mass gap would exist in the dual field theory [6]. The AdS/CFT correspondence gives the interpretation that the the supergravity modes represent excitations of particular gauge theory operators, which then possess a discrete mass spectrum. These results were verified in detail in refs. [7, 8].

Generalizing to $p+2$ dimensions, the AdS soliton metric may be written as

$$ds^2 = \frac{r^2}{L^2}\left(f(r)d\tau^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right) + \frac{L^2}{r^2}f^{-1}(r)dr^2\,, \qquad \text{with } f(r) = \left(1 - \frac{R^{p+1}}{r^{p+1}}\right), \tag{2.1}$$

where $\eta_{\mu\nu}dx^\mu dx^\nu$ is the p-dimensional Minkowski metric. This geometry can be constructed by a double analytic continuation of a planar AdS black hole in horospheric coordinates — see, for example, the discussion in ref. [13]. The geometry is only locally asymptotically AdS because implicitly the τ coordinate is chosen to be periodic in order to avoid a conical singularity at $r = R$. Choosing the period to be

$$\beta = \frac{4\pi L^2}{(p+1)R}\,, \tag{2.2}$$

the circle parametrized by τ smoothly shrinks to a point at $r = R$. As a result, in the context of supergravity, the fermionic fields and their dual gauge theory counterparts are antiperiodic around the τ circle.

This geometry was considered for the special cases $p = 3$ and 5 in ref. [7, 8] where the mass spectra for 0^{++} glueballs in QCD_3 and QCD_4 were calculated.[1] Ref. [7] also presents the mass spectra for 0^{--} and 0^{-+} glueballs by considering the appropriate supergravity fields. Scalar glueball spectra have also been analyzed in detail in refs. [20].

One proceeds by solving the linearized wave equation for a minimally coupled massless scalar field propagating in the above background (2.1). Specifically one considers the equation

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\,\phi\right) = 0\,. \tag{2.3}$$

In the case of $p = 3$, i.e. D3-branes, this scalar corresponds to the dilaton which is dual to the gauge theory operator which is the supersymmetric extension of $\mathrm{Tr}F^2$ [21, 2, 7]. One introduces the ansatz $\phi = b(r)e^{ik\cdot x}$, where $b(r)$ is the radial profile to be determined and the momentum k^μ is a p-vector with only components in the Minkowski space directions spanned by the coordinates x^μ. One then has $k^2 = -M^2$ giving the invariant mass-squared of the dual scalar operator in the effective p-dimensional field theory. Substituting this ansatz into eq. (2.3) one obtains an ordinary differential equation for the radial profile $b(r)$,

$$\frac{\partial^2 b(r)}{\partial r^2} + \frac{(p+2)r^{p+1} - R^{p+1}}{r\left(r^{p+1} - R^{p+1}\right)}\frac{\partial b(r)}{\partial r} + \frac{M^2L^2r^{p-2}}{r\left(r^{p+1} - R^{p+1}\right)}b(r) = 0\,. \tag{2.4}$$

To facilitate our analysis, this equation can be put into a Schrödinger-like form [10, 20] by redefining the wave function $b(r)$ as $b(r) = \beta(r)\chi(r)$ where,

$$\beta(r) = \sqrt{\frac{r-R}{r(r^{p+1} - R^{p+1})}}. \tag{2.5}$$

and then performing a change of variables according to $r = R(1+e^y)$. Eq. (2.4) now takes the form:

$$-\chi''(y) + V(y)\,\chi(y) = 0\,, \tag{2.6}$$

where the effective potential is given by

$$\begin{aligned} V(y) = \frac{1}{4} + \frac{e^{2y}\left(p(p+2)\left(1+e^y\right)^{2(p+1)} - 2p(p+2)\left(1+e^y\right)^{p+1} - 1\right)}{4\left(1+e^y\right)^2\left(\left(1+e^y\right)^{p+1} - 1\right)^2} - \\ - \frac{M^2L^4}{R^2}\frac{e^{2y}\left(1+e^y\right)^{p-3}}{\left(1+e^y\right)^{p+1} - 1}\,. \end{aligned} \tag{2.7}$$

[1] The discussion of $p = 5$ is rephrased there in terms of the throat geometry of D4-branes, using the duality between type IIa superstrings in ten dimensions and M-theory in eleven dimensions [19].

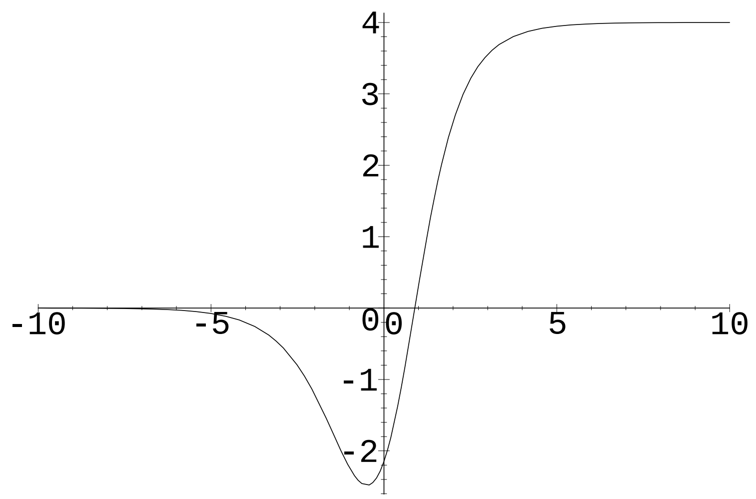

Figure 1: Plot of the effective potential $V(y)$ in eq. (2.7) for $p = 3$ and $\frac{M^2L^4}{R^2} = 50$. The figure demonstrates the existence of two classical turning points.

This complicated analytic form for the potential belies a relatively simple shape, as shown in figure 1. Now one must tune the potential by adjusting the parameter M^2 so that eq. (2.6) yields a normalizable bound state at *zero energy*. One thing that is easily verified is that the scalar field fluctuations produce no instabilities for this supergravity background. That is there are no tachyonic solutions since when M^2 is negative (or even positive but small — see below) the potential well in figure 1 disappears and $V(y)$ is everywhere positive.

The equation (2.6) can be solved either numerically [7], by matching asymptotic solutions [8] or in the WKB approximation [10, 20].[2] We will discuss here solving the spectrum for these scalar excitations in WKB approximation outlined in refs. [10, 20]. The potential given in eq. (2.7) takes the following asymptotic forms

$$
\begin{aligned}
V(y \gg 0) &= \frac{(p+1)^2}{4} - \frac{p(p+2)}{2} e^{-y} + \left(\frac{3p(p+2)}{4} - \frac{M^2L^4}{R^2}\right) e^{-2y} + \cdots \\
V(y \ll 0) &= \left(\frac{p+2}{4} - \frac{M^2L^4}{(p+1)R^2}\right) e^{y} + \cdots
\end{aligned}
\tag{2.8}
$$

Hence the classical turning point at large y is approximately

$$
y_+ = \log \frac{2ML^2}{(p+1)R}\,, \tag{2.9}
$$

[2]Note that the calculation described here for the AdS soliton in $p + 2$ dimensions is *not* the same as that in ref. [10]. There Minahan considers the propagation of the dilaton for a generalized Dp-brane background. Thus the results only agree with the present analysis for the special case $p = 3$.

where this result is valid to $O(M^{-1})$ in an expansion with $\frac{R}{ML^2} \ll 1$. The inner turning point is located at

$$y_- = -\infty\,. \tag{2.10}$$

as long as $M^2L^4/R^2 > (p+1)(p+2)/4$, which is consistent with the previous assumption. If the latter inequality is not satisfied, the potential is in fact everywhere positive and no bound states exist even without making the WKB approximation. In terms of the original r coordinate in eq. (2.1), these turning points correspond to,

$$r_+ = R + \frac{2}{p+1} ML^2 \qquad \text{and} \qquad r_- = R\,. \tag{2.11}$$

In the WKB approximation then, one finds a zero-energy bound state for

$$\left(n - \frac{1}{2}\right)\pi = \int_{y_-}^{y_+} \sqrt{V(y)}dy\,, \tag{2.12}$$

with n being a positive integer. Following refs. [10, 20], we expand the integral above as a series in powers of R/ML^2 and consider only terms appearing at $O(M)$ and $O(M^0)$. The corrections at the next order will appear at $O(M^{-1})$. Adding up the contributions and solving for M, one finds

$$M^2(p) = n\left(n + \frac{p-1}{2}\right)\frac{16\pi^3}{\beta^2}\left(\frac{\Gamma\left(\frac{p+3}{2(p+1)}\right)}{\Gamma\left(\frac{1}{p+1}\right)}\right)^2 + O(n^0)\,, \tag{2.13}$$

where β is the period of the compact coordinate τ given in eq. (2.2). For $p = 3$ and 5, which are relevant for $\mathrm{QCD}_{3,4}$, the above expression gives[3]

$$\begin{aligned} M^2(p=3) &\simeq \frac{56.67}{\beta^2} n\,(n+1) + O(n^0)\,, \\ M^2(p=5) &\simeq \frac{29.36}{\beta^2} n\,(n+2) + O(n^0)\,. \end{aligned} \tag{2.14}$$

3. The graviton spectrum

The purpose of the present paper is to repeat these calculations for gravitons propagating in the AdS_{p+2} soliton, and hence to calculate the spectrum of spin-two excitations for the effective p-dimensional field theories. The latter excitations are those associated with the stress-energy tensor $T_{\mu\nu}$ of the conformal field theory as this is the operator coupling to the AdS metric perturbations, i.e. the gravitons, according to the AdS/CFT correspondence [22, 23] — see discussion in ref. [18]. To determine the spectrum, we must solve the equations of motion for the gravitons on the background (2.1). Specifically we write the perturbed metric as

$$g_{ab} = \bar{g}_{ab} + h_{ab}\,, \tag{3.1}$$

[3]In comparing with the results of ref. [10], note that $M^2(\text{here}) = \pi^2 M^2(\text{there})$.

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where $\bar{g}_{ab}$ denotes the background metric (2.1) which is a solution of Einstein's equations in $p+2$ dimensions with a negative cosmological constant:

$$R_{ab} + \frac{p+1}{L^2} g_{ab} = 0\,. \tag{3.2}$$

Linearizing the latter equations of motion, the metric perturbation h_{ab} must satisfy [24]:

$$\frac{1}{2}\nabla_a\nabla_b\, h^c{}_c + \frac{1}{2}\nabla^2 h_{ab} - \nabla^c\nabla_{(a} h_{b)c} - \frac{p+1}{L^2} h_{ab} = 0\,, \tag{3.3}$$

where the background metric $\bar{g}_{ab}$ is used to raise and lower indices as well as to define the covariant derivatives in this equation. Now in analogy to the scalar field calculations, we make the ansatz that the graviton is given by $h_{ab} = H_{ab}(r)e^{ik\cdot x}$ where $H_{ab}(r)$ is the radial profile tensor and k^μ is a p-dimensional momentum vector with $k^2 = -M^2$. To determine the spectrum, we must solve eq. (3.3) with this ansatz as an eigenvalue problem for the mass M.

A feature here is the fact that $H_{ab}(r)$ is a tensor and there are thus a variety of polarizations, or graviton modes, that need to be considered. Further there are the ambiguities in the metric perturbations arising from diffeomorphism invariance, which we handle by imposing a "transverse gauge": $H_{a\mu}k^\mu = 0$. For massive excitations we may always, via the appropriate Lorentz boost, choose to work in the rest frame[4] so that the momentum can be written as $k^\mu = \omega\, \delta^\mu_t$. In this case, the transversality condition becomes,

$$H_{a\mu}k^\mu = 0 \;\Rightarrow\; H_{at} = 0\,, \qquad \forall a \tag{3.4}$$

Our implicit notation for the p-dimensional Minkowski space coordinates is $x^\mu = (t, x^i)$ with $i = 1, \ldots, p-1$.

3.1 Simple transverse traceless polarizations

In presenting the solutions of eq. (3.3), we begin with the gravitons polarized in the directions parallel to the hypersurface spanned by the coordinates x^μ, i.e. we give the solutions with

$$H_{\tau a} = H_{ra} = 0 = H_{a\mu}k^\mu \qquad \forall\, a\,, \tag{3.5}$$

where we have included the gauge condition (3.4) in this list of restrictions. From the point of view of the dual field theory, these excitations correspond to spin-two states. Other polarizations would be interpreted as scalar or vector states in the p-dimensional field theory — see below.

A consistent solution is provided by the following ansatz

$$H_{ab} = \varepsilon_{ab}\, \frac{r^2}{L^2} H(r)\,, \tag{3.6}$$

[4]Our analysis was done for the completely general case, however, here we anticipate a spectrum with positive definite mass squared.

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where the constant polarization tensor ε_{ab} satisfies the restrictions given in eq. (3.5). Solving the equations of motion (3.3) imposes one further restriction on the polarization, namely, it must be traceless

$$\eta^{\mu\nu}\varepsilon_{\mu\nu} = 0\,. \tag{3.7}$$

Thus eq. (3.6) describes $(p+1)(p-2)/2$ independent modes, which can be described as $(p-1)(p-2)/2$ off-diagonal polarizations, e.g.,

$$\varepsilon_{12} = \varepsilon_{21} = 1\,, \qquad \text{otherwise } \varepsilon_{ab} = 0 \tag{3.8}$$

and $(p-2)$ traceless diagonal polarizations, e.g.,

$$\varepsilon_{11} = -\varepsilon_{22} = 1\,, \qquad \text{otherwise } \varepsilon_{ab} = 0\,. \tag{3.9}$$

We also observe that $(p+1)(p-2)/2$ corresponds to precisely the number of polarizations of a massive spin-two particle in p dimensions.

For all of these independent polarizations, the radial profile $H(r)$ satisfies the same differential equation. Substituting the above ansatz (3.6) into eq. (3.3) yields

$$\frac{\partial^2 H(r)}{\partial r^2} + \frac{(p+2)r^{p+1} - R^{p+1}}{r(r^{p+1} - R^{p+1})}\frac{\partial H(r)}{\partial r} + \frac{M^2 L^4 r^{p-3}}{(r^{p+1} - R^{p+1})}H(r) = 0\,. \tag{3.10}$$

As for the case of the scalar equation (2.4), we put this equation into a Schrödinger-like form by setting $H(r) = \alpha(r)\phi(r)$ with

$$\alpha(r) = \sqrt{\frac{r - R}{r(r^{p+1} - R^{p+1})}}\,, \tag{3.11}$$

and by again changing variables according to $r = R(1 + e^y)$. Eq. (3.10) now takes the form,

$$-\phi''(y) + V(y)\phi(y) = 0\,, \tag{3.12}$$

where the effective potential is given by,

$$\begin{aligned} V(y) = \frac{1}{4} + \frac{e^{2y}\left(p(p+2)\left(1+e^y\right)^{2(p+1)} - 2p(p+2)\left(1+e^y\right)^{p+1} - 1\right)}{4\left(1+e^y\right)^2\left(\left(1+e^y\right)^{p+1} - 1\right)^2} - \\ - \frac{M^2 L^4}{R^2}\frac{e^{2y}\left(1+e^y\right)^{p-3}}{\left(1+e^y\right)^{p+1} - 1}\,. \end{aligned} \tag{3.13}$$

Now let us compare this effective potential (3.13) with eq. (2.7) for the minimally coupled massless scalar field considered in the previous section. One sees immediately that the effective potentials (2.7) and (3.13) are identical! In fact, the equations of motion (2.4) and (3.10) are also identical. In other words, the scalar and the transverse traceless gravitons considered here have exactly the same equations of motion

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and hence they have identical mass spectra, for all values of p. Via the AdS/CFT correspondence, this implies the degeneracy of tensor and scalar excitations in the corresponding dual field theories, i.e.

$$\frac{M_{2^{++}}}{M_{0^{++}}} = 1\,. \tag{3.14}$$

In the case of QCD_3, the prediction is that the glueball spectra of the operators TrF^2 and $T_{\mu\nu}$ are degenerate. We will comment more on this degeneracy in the discussion section.

Within the WKB approximation, we may read off the spectrum of the transverse traceless gravitons from the results of the scalar field analysis in eqs. (2.13) and (2.14). Note, however, that the scalar-tensor degeneracy is an exact statement, which holds outside of any approximation scheme used to compute the masses.

3.2 Exotic polarizations

The remaining linearized graviton solutions come in two categories. The first would appear as vectors in the dual field theory, and they may be given with the same ansatz as in eq. (3.6)

$$H_{ab} = \varepsilon_{ab}\,\frac{r^2}{L^2}H(r) \tag{3.15}$$

however, in this case the nonvanishing components of the polarization tensor take the form

$$\varepsilon_{\tau\mu} = \varepsilon_{\mu\tau} = v_\mu\,, \qquad \text{with } k\cdot v = 0 \text{ and } v\cdot v = 1\,. \tag{3.16}$$

Thus this vector solution contains $(p-1)$ independent modes. Substituting into the equations of motion (3.3) now yields the radial equation

$$\frac{\partial^2 H(r)}{\partial r^2} + \frac{(p+2)}{r}\frac{\partial H(r)}{\partial r} + \frac{M^2 L^4 r^{p-3}}{(r^{p+1}-R^{p+1})}H(r) = 0\,. \tag{3.17}$$

Using the field redefinition $H(r) = \alpha(r)\phi(r)$ where now $\alpha(r) = \sqrt{r - R/r^{p+2}}$ and again changing variables to the y coordinate as before, we find a Schrödinger-like equation where the potential is given by:

$$V(y) = \frac{1}{4} + \frac{p(p+2)e^{2y}}{4\,(1+e^y)^2} - \frac{M^2L^4}{R^2}\,\frac{e^{2y}\,(1+e^y)^{p-3}}{(1+e^y)^{p+1}-1}\,. \tag{3.18}$$

This potential has the following asymptotic forms,

$$\begin{aligned} V(y\gg 0) &= \frac{(p+1)^2}{4} - \frac{p(p+2)}{2}e^{-y} + \left(\frac{p(p+2)}{4} - \frac{M^2L^4}{R^2}\right)e^{-2y} + \cdots \\ V(y\ll 0) &= \frac{1}{4} - \frac{M^2L^4}{R^2}e^{y} + \cdots \end{aligned} \tag{3.19}$$

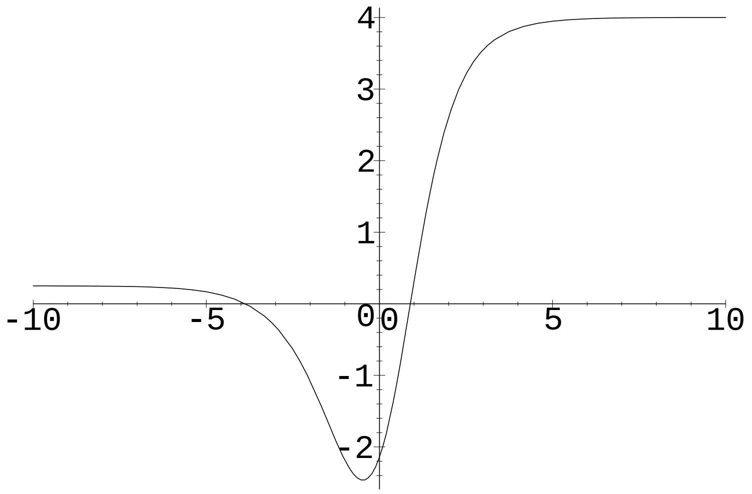

Figure 2: Plot of the effective potential $V(y)$ in eq. (3.18) for $p = 3$ and $\frac{M^2L^4}{R^2} = 50$. There are two classical turning points for finite values of y.

and thus has classical turning points located at:

$$\begin{aligned} y_+ &= \log \frac{2ML^2}{(p+1)R}, \\ y_- &= -\log \frac{4M^2L^4}{R^2(p+1)}. \end{aligned} \tag{3.20}$$

Note in this case that the inner turning point, r_- (given in terms of the original r coordinate), is situated away from the surface $r = R$. Given these results, the WKB mass spectrum is:

$$M^2(p) = n\left(n + \frac{p+5}{4}\right)\frac{16\pi^3}{\beta^2}\left(\frac{\Gamma\left(\frac{3+p}{2(p+1)}\right)}{\Gamma\left(\frac{1}{p+1}\right)}\right)^2 + O(n^0)\,. \tag{3.21}$$

Note that this WKB result is very similar to that for the scalars (and transverse traceless gravitons) given in eq. (2.13), with the only difference being in the coefficient of the $O(n)$ term. For the cases of interest for $\mathrm{QCD}_{3,4}$, this expression yields

$$\begin{aligned} M^2(p=3) &\simeq \frac{56.67}{\beta^2} n\,(n+2) + O(n^0) \\ M^2(p=5) &\simeq \frac{65.86}{\beta^2} n\left(n + \frac{5}{2}\right) + O(n^0) \end{aligned} \tag{3.22}$$

and so these vector excitations are slightly heavier than the tensors or scalars. The latter applies generally for $p < 7$, while in fact the vectors become lighter for $p > 7$. At $p = 7$, one finds that the vector spectrum here is degenerate with that for the scalars, however, this is a degeneracy which only holds within the approximations of our WKB calculation. Since the effective potentials (2.7) and (3.18) remain different for $p = 7$, this degeneracy should be lifted by higher order corrections, but we can not say which set of states will give the heavier spectrum.

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The last polarization is diagonal and would appear as a scalar excitation in the p-dimensional field theory. To construct this final solution of the linearized equations of motion (3.3), we begin with the following ansatz:

$$\begin{aligned} H_{\tau\tau}(r) &= -\frac{r^2}{L^2} f(r) H(r) \\ H_{\mu\nu}(r) &= \frac{r^2}{L^2}\left(\eta_{\mu\nu} a(r) + \frac{k_\mu k_\nu}{M^2}(b(r)+a(r))\right) \\ H_{rr}(r) &= \frac{L^2}{r^2} f^{-1}(r) c(r) \\ H_{r\mu}(r) &= ik_\mu\, d(r)\,, \end{aligned} \tag{3.23}$$

where $f(r)$ is as defined in eq. (2.1) and all other components of H_{ab} vanish. Note that we have not imposed the transverse gauge condition (3.4) in the ansatz above. Substituting this ansatz into eq. (3.3), consistency of the equations requires

$$a(r) = \frac{1}{p-1} H(r) \qquad c(r) = \frac{(p+1)R^{p+1}}{2pr^{p+1}-(p-1)R^{p+1}} H(r)\,, \tag{3.24}$$

while the function $d(r)$ is determined by $b(r)$ and $H(r)$ through the relation

$$\begin{aligned} d(r) = &-\frac{r^2}{2L^2M^2}\frac{\partial b(r)}{\partial r} + \frac{(p+1)r^2R^{p+1}}{2\left(2pr^{p+1}-(p-1)R^{p+1}\right)L^2M^2}\frac{\partial H(r)}{\partial r} + \\ &+ \frac{p(p+1)^2 r^{p+2}R^{p+1}H(r)}{\left(2pr^{p+1}-(p-1)R^{p+1}\right)^2 L^2M^2}\,. \end{aligned} \tag{3.25}$$

Hence one has the freedom to choose $d(r)$ by making the appropriate choice for the functional form of $b(r)$. For the present calculations, a convenient choice is to eliminate the derivative terms appearing in eq. (3.25), which allows one to express the polarization tensor explicitly in terms of $H(r)$. This gauge choice is accomplished by choosing

$$b(r) = -\frac{(p+1)R^{p+1}}{2pr^{p+1}-(p-1)R^{p+1}} H(r) \tag{3.26}$$

which yields

$$d(r) = \frac{2p(p+1)^2 r^{p+2}R^{p+1}H(r)}{M^2L^2\left(2pr^{p+1}-(p-1)R^{p+1}\right)^2}\,. \tag{3.27}$$

An alternative choice is to set $b(r) = -a(r)$, for which the diagonal components of the polarization along the x^μ directions have the form $\varepsilon_{\mu\nu} \propto \eta_{\mu\nu}$. Thus this choice makes manifest the Lorentz invariance of this polarization in the p dimensions of the field theory — otherwise it would only be Lorentz invariant up to a gauge transformation — and so it is obvious that these modes are actually dual to scalar excitations in the field theory.[5]

[5]Note that with $b = -a$ in eq. (3.23), there are still the off-diagonal components $H_{r\mu} \propto k_\mu$, but they do not constitute an independent vector. Further one could remove these components with an infinitesimal diffeomorphism with $v_a = -d(r)\exp(ik\cdot x)\,\delta^r_a$, but then the ansatz for the diagonal components becomes even more complicated.

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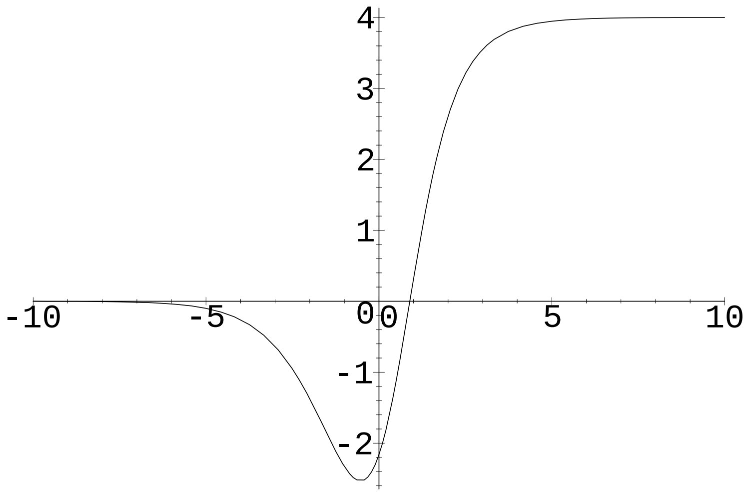

Figure 3: Plot of the effective potential $V(y)$ in eq. (3.30) for $p = 3$ and $\frac{M^2L^4}{R^2} = 50$. Note the similarity to fig. 1.

Using the results in eq. (3.24), the coupled set of equations (3.3) reduce to a single second order linear ODE for the function $H(r)$ which upon redefining $H(r)$ by $H(r) = \eta(r)\psi(r)$, where

$$\eta(r) = \sqrt{\frac{r - R}{r(r^{p+1} - R^{p+1})}} \tag{3.28}$$

and changing to the y coordinate of the previous sections takes the standard Schrödinger form (2.6) with a potential given by

$$\begin{aligned} V(y) = \frac{1}{4} + \frac{e^{2y}\left(p(p+2)\left(1+e^y\right)^{2(p+1)} - 2p(p+2)\left(1+e^y\right)^{p+1} - 1\right)}{4\left(1+e^y\right)^2\left(\left(1+e^y\right)^{p+1} - 1\right)^2} - \\ - \frac{M^2L^4}{R^2}\frac{e^{2y}\left(1+e^y\right)^{p-3}}{\left(1+e^y\right)^{p+1} - 1} - \frac{2(p-1)(p+1)^3\left(1+e^y\right)^{p-1}e^{2y}}{\left(\left(1+e^y\right)^{p+1} - 1\right)\left(2p\left(1+e^y\right)^{p+1} - p + 1\right)^2}. \end{aligned} \tag{3.29}$$

The potential is written here in a way so that it only differs from the scalar potential (2.7) by the addition of the last term.

In the asymptotic regions, the potential has the following forms

$$\begin{aligned} V(y \gg 0) &= \frac{(p+1)^2}{4} - \frac{p(p+2)}{2}e^{-y} + \left(\frac{3p(p+2)}{4} - \frac{M^2L^4}{R^2}\right)e^{-2y} + \cdots \\ V(y \ll 0) &= -\left((7p - 10) + \frac{M^2L^4}{(p+1)^2R^2}\right)e^{y} + \cdots \end{aligned} \tag{3.30}$$

As can be seen in eq. (3.30), the behavior for large positive y is identical to that given in eq. (2.8) thus the turning point is given by y_+ in eq. (2.9). However, the behavior for large negative y is slightly different but there is still a turning point at $y_- = -\infty$.

Thus, to leading order in the $1/M$ expansion [10] of the WKB integral (2.12), the mass spectrum is identical to that given in eq. (2.13),

$$M^2 = n\left(n + \frac{p-1}{2}\right)\frac{16\pi^3}{\beta^2}\left(\frac{\Gamma\left(\frac{3+p}{2(p+1)}\right)}{\Gamma\left(\frac{1}{p+1}\right)}\right)^2 + O(n^0)\,. \qquad (3.31)$$

The WKB mass spectra for these glueballs in $QCD_{3,4}$ is given by eq. (2.14). However, this degeneracy between the present scalar states and those in section 2 holds only within the WKB approximation. It will be lifted by higher order corrections.

4. Positive energy conjecture

In this section, we consider the implications of the above results for the positive energy conjecture put forth in ref. [13]. While the conjecture there was formulated for five dimensions, here we wish to consider the generalization to an arbitrary number of dimensions (greater than three). Hence consider solutions of Einstein's equations in $p+2$ dimensions with a negative cosmological constant, which asymptotically approach the AdS soliton metric $\bar{g}_{ab}$ given in eq. (2.1) with

$$g_{ab} = \bar{g}_{ab} + h_{ab}\,, \qquad (4.1)$$

where the perturbation h_{ab} has the following asymptotic behavior

$$h_{ab} = O(r^{-p+1})\,, \qquad h_{ar} = O(r^{-p-1})\,, \qquad h_{rr} = O(r^{-p-3})\,, \qquad \text{with } a, b \neq r\,. \quad (4.2)$$

Then $E(g_{ab}) \geq E(\bar{g}_{ab})$ with equality obtained if and only if $g_{ab} = \bar{g}_{ab}$. In ref. [13], it was verified that, for the case of $p = 3$, the AdS soliton is perturbatively stable to all linearized deformations of the metric. In the following, we apply directly the results obtained in section 3 to give a proof of perturbative stability of this solution in arbitrary numbers of dimensions.

The essential point is that in the spectra calculated in the previous section, we found that $M^2 > 0$ in all cases. Hence through the AdS/CFT correspondence, these metric fluctuations will be dual to positive energy excitations of the field theory, with $E = \sum n_i\,\omega_i$ where n_i is the number of quanta excited in a particular mode with frequency $\omega = k^t$. From this point of view, negative energies or instabilities would be signaled by the appearance of "tachyonic" excitations with $M^2 < 0$. For a complete discussion of perturbative stability of the AdS soliton, one should also consider all linearized solutions including those with momenta in the compact τ direction. Naively, one expects that adding such internal momenta will only increase the masses. In fact, this intuition is correct, and we leave the detailed analysis of these modes to the appendix. Hence $M^2 > 0$ for all graviton modes, and so this indicates that the AdS soliton is stable against all linearized metric fluctuations in any number of dimensions.

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Given that we wish to determine whether a particular solution of Einstein's equations is perturbatively stable against metric deformations, we should be able to address this question in a purely gravitational framework, without reference to the AdS/CFT correspondence. We do so now by considering the construction for gravitational energy by Abbot and Deser [25]. One begins by dividing the metric as in eq. (4.1), but now the metric deformation h_{ab} is considered to be defined globally. Given this decomposition and the fact that the background metric $\bar{g}_{ab}$ satisfies Einstein's equations with a negative cosmological constant,

$$\bar{R}_{ab} - \frac{1}{2}\bar{g}_{ab}\bar{R} - \frac{p(p+1)}{2L^2}\bar{g}_{ab} = 0\,, \tag{4.3}$$

the equations determining the deformation h_{ab} may be written as [25]:

$$R^L_{ab} - \frac{1}{2}\bar{g}_{ab}R^L + \frac{p(p+1)}{2L^2}h_{ab} = T_{ab}\,. \tag{4.4}$$

Here the superscript L denotes that these curvature terms have been linearized in h_{ab}, while the "stress-energy tensor" on the right-hand side is defined to include all terms second order and higher in h_{ab}. Because the left-hand side of eq. (4.4) obeys the background Bianchi identity, $\bar{\nabla}^a\left(R^L_{ab} - 1/2\bar{g}_{ab}R^L + p(p+1)/(2L^2)h_{ab}\right) = 0$, the Einstein equations (4.4) dictate that

$$\bar{\nabla}^a T_{ab} = 0 \tag{4.5}$$

as an exact result, where $\bar{\nabla}_a$ denotes the covariant derivative with respect to the background metric $\bar{g}_{ab}$. Further, it is understood that all indices are raised and lowered here using the background metric. Now, given a Killing vector ξ^a of the background metric, one has

$$\bar{\nabla}^a\left(T_{ab}\xi^b\right) = 0 \tag{4.6}$$

as a result of the Killing vector identity, $\bar{\nabla}_a\xi_b + \bar{\nabla}_b\xi_a = 0$, and the symmetry of T_{ab}. Denoting the one-form

$$(T\cdot\xi) = dx^a\,T_{ab}\xi^b\,, \tag{4.7}$$

then the dual $(p+1)$-form is closed, i.e.

$$d*(T\cdot\xi) = 0\,, \tag{4.8}$$

given the vanishing divergence in eq. (4.6). Thus integrating this form over a $p+1$-dimensional surface with a time-like normal

$$\int *(T\cdot\xi) \tag{4.9}$$

yields a conserved charge.[6] Now focusing our attention on the time-like Killing vector $\xi^a\partial_a = \partial_t$ of the AdS soliton background, the Killing energy is defined as the

[6] Note that this integral is finite given the rate at which h_{ab} vanishes at asymptotic infinity in eq. (4.2).

conserved charge

$$E = \frac{1}{8\pi G}\int *(T\cdot\xi)\,, \tag{4.10}$$

where the normalization is chosen here to match that of alternate definitions for asymptotically AdS spacetimes, such as in refs. [18, 26, 27, 28].

The preceding discussion refers to an exact solution of Einstein's equations, i.e.given the metric decomposition (4.1) with the background satisfying eq. (4.3), eq. (4.4) is satisfied to all orders in h_{ab}. In the analysis of the previous section, we have only been considering metric deformations which satisfy the linearized Einstein equations (3.3), i.e. all of the higher order terms implicit in T_{ab} in eq. (4.4) have been dropped. Consistent with this linearized analysis, our strategy will be to identify the stress-energy density of our perturbed spacetimes to be the contributions quadratic in h_{ab} in eq. (4.10). This approach will be sufficient to identify whether or not the background metric is *perturbatively* stable.

It is simplest to apply this analysis to the transverse traceless modes of section 3.1. We write the metric perturbation as

$$h_{ab} = \varepsilon_{ab}\frac{r^2}{L^2}H(r,\tau,x^\mu)\,, \tag{4.11}$$

where the constant polarization tensor satisfies the constraints $\varepsilon_{ra} = \varepsilon_{\tau a} = 0 = \varepsilon_{a\mu}\nabla^\mu H$, as well as $\eta^{\mu\nu}\varepsilon_{\mu\nu} = 0$. Also the polarization will be normalized such that $\varepsilon^{ab}\varepsilon_{ab} = 2$, as in the examples in section 3. Inserting this ansatz into eq. (4.10) and keeping only the quadratic terms, one finds

$$E = \frac{1}{32\pi G}\int dr\,d\tau\,d^{p-1}x\,\left(\frac{r}{L}\right)^{p-2}\left((\partial_t H)^2 - H\partial_t^2 H\right), \tag{4.12}$$

where we integrated by parts in both r and τ, and used the linearized equations of motion to produce this simple expression. Substituting for eq. (4.11) the real part of the solutions found in section 3.1 or in the appendix, i.e. $H(r,\tau,t,x^i) = H(r)\,\cos(k\cdot x + q\tau)$, yields

$$E = \frac{\omega^2}{32\pi G}\int dr\,d\tau\,d^{p-1}x\,\left(\frac{r}{L}\right)^{p-2},H(r)^2 \tag{4.13}$$

where $\omega = k^t$. Clearly now, this expression is manifestly positive definite, and so the AdS soliton is perturbatively stable to deformations by these modes. Note that this classical result is consistent with that stated above from the AdS/CFT correspondence. In particular, the Lorentz-invariant measure for momentum modes is $d^{p-1}k/\omega(\vec{k})$ which is why the field theory energy is proportional to ω while the classical result (4.13) has a factor of ω^2. Further the quantum number operator in the field theory will be proportional to the field squared as in eq. (4.13), and the factor of G^{-1} would be absorbed using the canonical normalization for the quantum graviton field — see, e.g., ref. [30]. Note that here we assume a simple local mapping between

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the perturbative quantum field in the supergravity theory, i.e. the gravitons, and the dual quantum field in the conformal field theory, as in the discussions of ref. [31] — see below, however. Performing the same analysis for the exotic polarizations of section 3.2 and appendix A is more complicated. The results are simple, however, if we consider only the modes which are independent of the compact coordinate τ. For example, consider our ansatz for the vector modes

$$h_{ab} = \varepsilon_{ab} \frac{r^2}{L^2} H(r) \cos k \cdot x \,, \tag{4.14}$$

where the nonvanishing components of the polarization tensor are $\varepsilon_{\tau\mu} = \varepsilon_{\mu\tau} = v_\mu$ with $k \cdot v = 0$ and $v \cdot v = 1$. Note that in order to produce a sensible energy, we have again taken the real part of the ansatz used in section 3. Such a metric perturbation then yields

$$E = \frac{\omega^2}{32\pi G} \int dr d\tau\, d^{p-1}x \left(\frac{r}{L}\right)^{p-2} f^{-1}(r)\, H(r)^2 \,, \tag{4.15}$$

where $f(r)$ is defined in eq. (2.1), and $\omega = k^t$ as above. Similarly if we take the real part of the ansatz presented in eq. (A.10) for the scalar modes, we find a relatively simple expression for the quadratic Killing energy

$$E = \frac{p}{p-1} \frac{\omega^2}{64\pi G} \int dr d\tau\, d^{p-1}x \left(\frac{r}{L}\right)^{p-2} H(r)^2 \,. \tag{4.16}$$

Both of these expressions are manifestly positive definite, and so obviously the AdS soliton is perturbatively stable to deformations by these modes.

The gravitational calculation of the energy of the vector and scalar modes becomes far more complicated. For example, for the vector modes with τ-dependence, one works with the real part of the ansatz described in eq. (A.5). Inserting this into the above formulae (4.10) yields an extremely long expression for the energy density. One can arrange this expression in powers of the AdS radius L giving,

$$E = \frac{M^2}{64\pi G} \int dr d\tau\, d^{p-1}x \left(\frac{r}{L}\right)^{p-2} \left\{ \frac{r^{p+1}\left(1+q^2/M^2\right)^2}{r^{p+1}\left(1+q^2/M^2\right) - R^{p+1}} H(r)^2 + O\left(\frac{q^2}{L^4}\right) \right\} , \tag{4.17}$$

where the remaining implicit terms at $O(q^2/L^4)$ and $O(q^4/L^8)$ fill another page and a half, and are not particularly illuminating. In particular, while one can see immediately that the first term above is positive definite, the sign of the remaining higher order contributions is not obvious. Therefore establishing that the energy in eq. (4.17) is positive definite requires a more extended analysis. We will not present such an analysis here, but one can be confident in the result since there is also the much simpler proof relying on the AdS/CFT duality described above. One also runs into similar complications in calculating the gravitational energy for the scalar modes with nontrivial τ-dependence.

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As a final point on the discussion of the gravitational energy, it may be disconcerting that the expressions for the vector and scalar modes with nontrivial τ-dependence, e.g., eq. (4.17), are so complicated. These expressions suggest that there is a complicated nonlocal relationship between the quantum operators producing the graviton excitations in the AdS soliton background and the dual operators in the field theory. This result may be surprising given that in ref. [31], a simple identity was established between the AdS and CFT quantum modes for all excitations. However, that analysis applied for a (super)gravity background which was purely anti-de Sitter space, and the identification relied on the large symmetry group of this background. Only a small part of this symmetry group survives for the AdS soliton background, and so perhaps it not so surprising that the relationships can become more complicated. These complications are probably also related to the nonlocal expressions that arise for various metric components in the relevant graviton modes, e.g., see eq. (A.6).

5. Discussion

In this paper we have calculated the complete spectrum of metric fluctuations of the $(p+2)$-dimensional AdS soliton given in eq. (2.1). The dual conformal field theory will be formulated on a $(p+1)$-dimensional background geometry $M^p \times S^1$ with metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + d\tau^2 \tag{5.1}$$

where τ inherits the same period β as in eq. (2.2). In the limit, that this circle direction is small, the field theory should become effectively p-dimensional. From string theory, it is in this situation that the AdS soliton has been suggested to give a dual description of ordinary Yang-Mills theory in three or four dimensions. For the polarizations which are confined to the x^μ directions then, our results correspond to the mass spectrum of the spin-two excitations of the effective p-dimensional field theory. A second interesting implication of our analysis is that since we have found that $M^2 > 0$ for all of the modes, our results confirm the perturbative stability of the $(p+2)$-dimensional AdS soliton, and thus provide evidence the positive energy conjecture of ref. [13] can be extended to arbitrary numbers of dimensions.

Being a massless spin-two field, the graviton propagating in any $(p+2)$-dimensional spacetime has $\frac{1}{2}p(p+1) - 1$ physical degrees of freedom. In the present case, these degrees of freedom are organized as representations of the p-dimensional Lorentz symmetry of the AdS soliton background. These are: $(p+1)(p-2)/2$ modes in massive spin-two representations, $p-1$ polarizations in massive vector representations, and 1 set of massive scalar states. Each of these sets of states is further labeled by the integer $\tilde{n}$ giving the momentum in the τ-direction, i.e. $q = 2\pi\tilde{n}/\beta$. These graviton modes are dual to operators in the corresponding conformal field theory with the same quantum numbers. In fact, one knows that the relevant operator is the stress-energy tensor T_{ab} of the $(p+1)$-dimensional field theory as this is the operator coupling to the AdS metric perturbations according to the AdS/CFT correspondence [22, 23]. Of course, the various components of T_{ab} are decomposed according to the $\mathrm{SO}(1,p-1)$ symmetry of the boundary manifold (5.1) matching the decomposition of the metric fluctuations described above.

Thus when the various graviton modes are excited in the AdS soliton background, there should be a corresponding excitation in the expectation value of the stress-energy tensor in the dual field theory description. One can verify that the metric fluctuations fall off with precisely the appropriate rate to yield a nonvanishing $\langle T_{ab}\rangle$, i.e. $h_{ab} = O(r^{-p+1})$ in $p+2$ dimensions [18]. Examining the asymptotic behavior of the effective potential for any of the modes, one finds

$$V(y \gg 0) \simeq \frac{(p+1)^2}{4} + \cdots \tag{5.2}$$

Thus to leading order in this regime, the normalizable solution of the Schrödinger equation behaves as

$$\psi(y \gg 0) \simeq e^{-\frac{p+1}{2}y} \simeq \left(\frac{R}{r}\right)^{\frac{p+1}{2}} . \tag{5.3}$$

Now while the details of the field redefinition factor varies from mode to mode, one finds that they all have the same asymptotic behavior at large r, namely $\eta(r >> 0) \simeq (R/r)^{\frac{p+1}{2}}$, and hence to leading order, the radial profile becomes

$$H(r) = \eta(r)\psi(r) \simeq \left(\frac{R}{r}\right)^{p+1} . \tag{5.4}$$

Finally various components of the metric fluctuations contain $r^2 H(r) = O(r^{-p+1})$, giving precisely the desired fall off.

So for example, the transverse traceless gravitons correspond to inducing an excitation of the stress-energy in the field theory with

$$\langle T_{\mu\nu}\rangle \propto \varepsilon_{\mu\nu} \cos(k\cdot x + q\tau) , \tag{5.5}$$

where ε is the polarization tensor appearing in the metric fluctuation which satisfies $\varepsilon_{\mu\nu}k^\nu = 0 = \eta^{\mu\nu}\varepsilon_{\mu\nu}$. Similarly for the vector modes, one finds from eq. (A.5) that the nonvanishing stress-energy components are

$$\langle T_{\tau\mu}\rangle \propto v_\mu \cos(k\cdot x + q\tau) \qquad \text{and} \qquad \langle T_{\mu\nu}\rangle \propto \frac{q}{M^2}(k_\mu v_\nu + v_\mu k_\nu)\cos(k\cdot x + q\tau) \tag{5.6}$$

where $k\cdot v = 0$ and $v\cdot v = 1$. Here, although there are nontrivial metric components $h_{r\mu}$, asymptotically they fall off too rapidly to contribute in the calculation of $\langle T_{ab}\rangle$ [18]. Finally for the scalar mode (3.23) with $q = 0$, one finds

$$\langle T_{\tau\tau}\rangle \propto -(p-1)\cos(k\cdot x) \qquad \text{and} \qquad \langle T_{\mu\nu}\rangle \propto \left(\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\right)\cos(k\cdot x) \tag{5.7}$$

Note that all of these stresses are traceless and transverse in the $p+1$-dimensional background (5.1) of the field theory, i.e. $\langle T^a{}_a \rangle = 0 = \langle T_{ab} \rangle k^b$ where $k^a = (k^\mu, q)$. Note that the three expressions above correspond to the additional stress-energy associated with the graviton. The AdS soliton itself also induces an nontrivial Casimir stress-energy [18].

Note that with a single mode excited in its rest frame, that is with $k^a = (M, 0, \ldots, 0, q)$, no energy density is induced. There are only stresses in the spatial directions for all such modes. However, in general there is a nontrivial expectation $\langle T_{tt} \rangle$, and in fact there are regions of negative as well as positive energy density because of the cosine factors. Note however that the negative and positive contributions precisely cancel out to yield zero total energy at this order. For example, in the cases where $q \neq 0$, integrating over the compact τ-direction is enough to produce a vanishing result. This is the expected result since the induced energy densities here are linear in the amplitude of the metric fluctuations, and we found in section 4 that the total energy was in fact quadratic in this amplitude. Similar observations were made in ref. [32] where the scattering of gravitons in AdS space was considered. The vanishing total energy emerging from these calculations is a result of the fact that we have only considered the linearized Einstein equations in discussing the present graviton modes. If one were to completely solve the full Einstein equations or at least solve them to next order in the amplitude of the waves, one would find that the long-range metric perturbations would receive second-order contributions yielding $E = \int d\tau d^{p-1}x^i \, \langle T_{tt} \rangle > 0$. The integrand of the energy expressions in section 4 would be closely related to the source terms for the second order contribution in the metric fluctuation h_{tt}. These second order contributions would ensure that while there are locally regions with negative energy density, they are always accompanied by regions with a larger net positive energy density, in keeping with the quantum interest conjecture [33].

Since the vector modes couple to τ-components of the stress-energy tensor (even with $q = 0$) in the field theory, as shown in eq. (5.6), and the background metric (5.1) is diagonal, one would require that these modes should decouple if the AdS soliton is to provide a good description of an effective p-dimensional field theory.[7] In fact, however, there is no evidence of such decoupling[8] as the masses of all states are roughly of the order β^{-1}. In fact, as noted before, the vector modes are lighter than the spin-two modes for $p > 7$. This is similar to the non-decoupling of excitations on the internal S^5 found in considering the case of $p = 3$ [29]. In this string theory context, one might consider the effect of α'-corrections to the string theory action [34].

[7] The same should be true for the scalar modes. However, with $q = 0$ only $\langle T_{\tau\tau} \rangle \neq 0$ in eq. (5.7), and this term could arise from a contribution proportional to the background metric (5.1) which is diagonal.

[8] In fact, we already see that the theory really fails to become p-dimensional as the lightest excitations have masses of the order of the compactification scale.

Such higher derivative terms would modify both the AdS soliton background [35] and the equations of motion. However, we did not pursue these lengthy calculations. In the context of the higher S^5 harmonics, it was found that the leading α'-corrections did not produce the desired decoupling, and it was conjectured that decoupling may only result through nonperturbative effects [29].

A surprising result that we have found is that for all values of p the spectrum of the spin-two modes is exactly degenerate with that of a minimally coupled massless scalar field. In the dual field theory, there will be an exact degeneracy of the corresponding spin-two and spin-zero excitations. In particular then, in the case of $p = 3$ where the effective field theory is ordinary (i.e. nonsupersymmetric) Yang-Mills theory in three dimensions [6], there will be a degeneracy of the spin-two glueballs associated with the stress tensor $T_{\mu\nu}$, and the spin-zero glueballs associated with the operator $\mathrm{Tr}F^2$. We emphasize that the degeneracy of these spectra is a consequence of the fact that the dilaton and the gravitons satisfy precisely the same equation of motion, and is not an artifact of the WKB calculations presented in section 3. There is a similar degeneracy for spin-two and scalar glueballs in QCD_4, which is associated with the AdS_7 soliton with $p = 5$ [6] — see below. This is a surprising prediction of the AdS/CFT correspondence as current lattice simulations [14, 15, 16] certainly do not provide any evidence of such a remarkable degeneracy. Rather the lattice calculations suggest that the spin-two excitations should be more massive than the scalar glueballs. For the lowest lying glueballs, simulations for QCD_3 [14] estimate $M_{2^{++}}/M_{0^{++}} \simeq 1.65 \pm 0.04$ for the large N limit. Similarly, for QCD_4 [15], lattice calculations suggest $M_{2^{++}}/M_{0^{++}} \simeq 1.39 \pm 0.13$ in the large N limit. (The errors quoted are the statistical errors arising in extrapolating to the continuum from the lattice.) Of course, this discrepancy between the lattice calculations and the AdS/CFT correspondence is not a direct contradiction. The gravity calculations performed here would be valid for $\mathrm{SU}(N)$ gauge theory in the large N limit and for (very) large 't Hooft coupling g^2N. In contrast the lattice results would hold for weak 't Hooft coupling. Hence it might be that this degeneracy is a result of the strong coupling limit relevant for the supergravity calculations. Again, it would be interesting to see to what extent the degeneracy is lifted by α'-corrections to the low energy action. It would also be interesting to see if this degeneracy survives in other supergravity models exhibiting QCD-like behavior [11, 12] (such as running couplings and asymptotic freedom).

We have no insight as to why the spectrum of the spin-two graviton modes should be degenerate with that of a scalar field in general. However, for $p = 3$, it can be related to previous observations about D3-branes. In ref. [22], it was found that in the background geometry of a D3-brane, the gravitons polarized along the world-volume had an action of the form,

$$I = -\frac{1}{64\pi G_{10}} \int d^{10}x\sqrt{-g}\partial_\mu h_{ab}\partial^\mu h_{ab} \tag{5.8}$$

and thus, from the point of view of the *transverse* space and time, they behave as minimally coupled massless scalars. As a result, it was shown that the classical absorption cross-sections were identical for these gravitons and the dilaton. Now it is easy to verify that the same observations still hold true in the geometry of an analytically continued near-extremal D3-brane, as long as the graviton polarizations have no τ-components. The throat geometry of such a D3-brane would be the direct product of the AdS_5 soliton with a five-sphere. The AdS_5 soliton geometry is comprised of the world-volume directions and the radial direction in the transverse space. The matching of the graviton and the scalar field equations of motion has also been observed in refs. [32, 39]. In the spectrum calculations on the AdS soliton background, one focuses essentially on the time and radius part of the wave equation, which would be identical to that arising in the propagation of an S-wave in the throat geometry. Thus one can interpret the result, that the field equations governing the spin-two modes are the same as for a scalar field, as a reflection of the above observation that these spin-two modes behave as scalar fields in the transverse space.

In ref. [10], Minahan calculated the spectrum of the dilaton (a minimally coupled massless scalar) propagating in the throat geometry of an analytically continued near-extremal Dp-brane for general p. These throat geometries are not precisely the same as the AdS_p soliton as these backgrounds also contain a nontrivial dilaton field, except for $p = 3$. One may note, however, that there is a matching of his dilaton spectrum for the D1,3,4-branes with that of a scalar in the AdS soliton with $p = 2, 3, 5$, respectively. Of course, this is not a surprise for the D3-brane where the throat geometry is just the AdS soliton with $p = 3$. For the D4-brane, the matching arises because this brane in the type IIa string theory can be lifted to an M5-brane in D=11 supergravity [36], where the throat geometry contains an AdS_7 soliton factor with $p = 5$. Similarly through a chain of dualities, the D1-brane of the type IIb string theory can be related to the M2-brane, where the AdS_2 soliton with $p = 2$ appears in the throat geometry. However, note that there are no scalar fields in D=11 supergravity. The string theory dilaton arises as part of the eleven-dimensional metric. Hence the matching of the spectra that we noted above only occurs because of the degeneracy of spin-two graviton modes and scalar field excitations in the AdS soliton background, since Minahan [10] actually calculated the spectrum of a particular mode of the graviton in a dual description. In fact, in the case of the M2-brane for which there are no spin-two excitations, the degeneracy arises as an artifact of the WKB approximation for which the spin-zero graviton spectrum (3.31) matched that of the scalar field (2.13).

The case of the M5-brane is of particular interest as it may give a dual description of QCD_4 [6]. One can regard the field theory as being obtained from the compactification of the six-dimensional $(0, 2)$ conformal field theory on $S^1 \times S^1$ where the first circle is much smaller than the second circle, on which one also imposes supersymmetry breaking boundary conditions [6]. In terms of the AdS/CFT correspondence, this is equivalent to M-theory on an AdS soliton with $p = 5$ in a direct product with a four-sphere. The AdS soliton geometry is

$$ds^2 = \frac{r^2}{L^2}\left(f(r)d\tau^2 + dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right) + \frac{L^2}{r^2}f^{-1}(r)dr^2 \qquad \text{with } f(r) = 1 - \frac{R^6}{r^6} \quad (5.9)$$

where z and τ are the coordinates parameterizing the first and second circles above, respectively. As described above, dimensionally reducing on z yields type IIa string theory on the throat geometry of a(n appropriate) D4-brane. Now with respect to the x^μ-directions, there are two spin-zero modes in the metric fluctuations. The first would be the polarization identified as the scalar mode given in eq. (3.23) with $p = 5$. The second would be a mode identified as a transverse traceless mode in section 3.1 with a polarization tensor of the form

$$\varepsilon_{zz} = -4 \qquad \text{and} \qquad \varepsilon_{\mu\nu} = \eta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\,. \quad (5.10)$$

As both of these modes would contain h_{zz} fluctuations, they would both seem to mix with the ten-dimensional string theory dilaton [37]. We are not sure how these two modes are distinguished, but presumably one should decouple while the other is dual to spin-zero glueballs associated with the operator $\mathrm{Tr}F^2$ in QCD_4. In any event, at least with in the WKB approximation, the spectra of both modes are degenerate, and so our calculations suggest a degeneracy of the spin-two and spin-zero glueballs at least in the large N limit of QCD_4.

Acknowledgments

This research was supported by NSERC of Canada and Fonds FCAR du Québec. We would like to acknowledge useful conversations with Gary Horowitz, Guy Moore and Soo-Jong Rey. RCM would also like to thank the Institute for Theoretical Physics at UCSB for its hospitality in the final stages of this work. While at the ITP, RCM was supported by NSF Grant PHY94-07194.

We have been informed that S. Lee, J. Park, S. Moon and S.-J. Rey [38] have also considered the graviton spectra for general Dp-brane backgrounds, and had observed the degeneracy of the spectra for the dilaton and transverse traceless gravitons on the AdS_5 soliton.

A. Appendix

In this appendix, we provide some of the details for the analysis of the metric fluctuations with nontrivial τ-dependence. In this case the ansatz made in section 3 is extended to $h_{ab} = H_{ab}(r)e^{i(k\cdot x+q\tau)}$ where q is the momentum in the compact direction given by $q = 2\pi\tilde{n}/\beta$ for any integer $\tilde{n}$ and the period β is given in eq. (2.2).

For the purposes of gauge fixing, it is natural to include q as the last component of a $(p+1)$-dimensional momentum vector, $k^a = (k^\mu, q)$. For the sake of simplicity, we will choose to work in the p-dimensional "rest frame" of these excitations, i.e.with an appropriate Lorentz boost, we set $k^a = (M, 0, \ldots, 0, q)$. Now the natural transversality condition to impose on the metric perturbations becomes

$$H_{ab}k^b = 0 \Rightarrow H_{at} = -\frac{q}{M} H_{a\tau}\,. \tag{A.1}$$

We begin with extension of the simple transverse traceless polarizations. As in eq. (3.6), we set

$$H_{ab} = \varepsilon_{ab}\frac{r^2}{L^2}H(r)\,, \tag{A.2}$$

where $\varepsilon_{\tau a} = \varepsilon_{ra} = \varepsilon_{ta} = 0$. Substituting the extended ansatz into eq. (3.3), one finds that the only effect of adding the compact momenta is to shift the mass term in eq. (3.10) by $M^2 \to M^2 - q^2/f(r)$ where $f(r)$ is the function appearing in the metric in eq. (2.1). After performing the same wavefunction redefinition and change of variables as described in section 3.1, the equation of motion becomes Schrödinger-like (3.12) with an effective potential

$$\begin{aligned} V(y) = \frac{1}{4} + \frac{e^{2y}\left(p(p+2)\left(1+e^y\right)^{2(p+1)} - 2p(p+2)\left(1+e^y\right)^{p+1} - 1\right)}{4\left(1+e^y\right)^2\left(\left(1+e^y\right)^{p+1} - 1\right)^2} - \\ - \frac{M^2L^4}{R^2}\frac{e^{2y}\left(1+e^y\right)^{p-3}}{\left(1+e^y\right)^{p+1} - 1} + \frac{q^2L^4}{R^2}\frac{e^{2y}\left(1+e^y\right)^{2p-2}}{\left((1+e^y)^{p+1} - 1\right)^2}\,. \end{aligned} \tag{A.3}$$

The latter has the following asymptotic behavior

$$\begin{aligned} V(y \gg 0) &= \frac{(p+1)^2}{4} - \frac{p(p+2)}{2}e^{-y} + \left(\frac{p(p+2)}{4} - \frac{M^2L^4}{R^2}\right)e^{-2y}\,, \\ V(y \ll 0) &= \frac{\tilde{n}^2}{4} + \left(\frac{p+2}{4} - \frac{M^2L^4}{(p+1)R^2} + \frac{\tilde{n}^2}{4}\right)e^y. \end{aligned} \tag{A.4}$$

Note that the turning point for $y \ll 0$ is now at a finite coordinate distance due to the inclusion of a non-zero compact momenta. This has the anticipated effect of shifting the mass spectrum upwards from that given in eq. (2.13). In any event, one can verify that for $M^2 < 0$ the potential is everywhere positive and so the Schrödinger equation yields no normalizable zero-energy solutions. Hence, since the spectrum has $M^2 > 0$ for all of these spin-two modes, they clearly do not present any difficulty for the stability of the AdS soliton. Finally, we also note that the above results are again identical to that for the massless, minimally coupled scalar in the case that internal momentum is included. The the spectra of these two set of modes remains identical when one allows for nontrivial τ-dependence.

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Next we consider the vector modes with nontrivial τ-dependence. In this case, the ansatz for the polarization of eq. (3.15) had to be modified as well. Using the transverse gauge condition (A.1) in the rest frame, a consistent ansatz is given by

$$H_{\tau i} = \frac{r^2}{L^2} H(r)\,, \qquad H_{ti} = -\frac{q}{M}\frac{r^2}{L^2} H(r)\,, \qquad H_{ri} = -\frac{q}{M}\frac{r^2}{L^2} a(r)\,, \tag{A.5}$$

for some $i \in \{1, \ldots, p-1\}$, and all other components of H_{ab} are zero. Consistency of this ansatz in the equations of motion (3.3) requires that

$$a(r) = \frac{R^{p+1}}{r^{p+1}\left(1+q^2/M^2\right) - R^{p+1}} \frac{1}{M} \frac{\partial H(r)}{\partial r}\,. \tag{A.6}$$

Redefining the radial profile according to $H(r) = \alpha(r)\phi(r)$ with

$$\alpha = \sqrt{\frac{r-R}{r^{p+2}} \frac{r^{p+1}\left(1+q^2/M^2\right) - R^{p+1}}{r^{p+1} - R^{p+1}}} \tag{A.7}$$

and then making the usual change of variables, $r = R(1+e^y)$, leads to an effective potential of the form,

$$\begin{aligned} V(y) = {} & \frac{1}{4} + \frac{p(p+2)e^{2y}}{4\left(1+e^y\right)^2} - \frac{M^2 L^4}{R^2} \frac{e^{2y}\left(1+e^y\right)^{p-3}\left(\left(1+e^y\right)^{p+1}\left(1+q^2/M^2\right) - 1\right)}{\left(\left(1+e^y\right)^{p+1} - 1\right)^2} - \\ & - \frac{q^2}{M^2} \frac{p(p+2)\left(\left(1+e^y\right)^{p+1}\left(2+q^2/M^2\right) - 2\left(3+q^2/M^2\right)\right)\left(1+e^y\right)^{3p+3} e^{2y}}{4\left(1+e^y\right)^2 \left(\left(1+e^y\right)^{p+1} - 1\right)^2 \left(\left(1+e^y\right)^{p+1}\left(1+q^2/M^2\right) - 1\right)^2} + \\ & + \frac{q^2}{M^2} \frac{\left(1+e^y\right)^{2p+2}\left(4+q^2/M^2\right) + 4\left(1+e^y\right)^{p+1} e^{2y}}{4\left(1+e^y\right)^2 \left(\left(1+e^y\right)^{p+1} - 1\right)^2 \left(\left(1+e^y\right)^{p+1}\left(1+q^2/M^2\right) - 1\right)^2}\,. \end{aligned} \tag{A.8}$$

Note that this is nearly identical to eq. (3.18) except for the last term which is proportional to q^2. This effective potential has the following asymptotic behavior,

$$\begin{aligned} V(y \gg 0) &= \frac{(p+1)^2}{4} - \frac{p(p+2)}{2} e^{-y} + \left(\frac{p(p+2)}{4} - \frac{M^2 L^4}{R^2}\left(1+q^2/M^2\right)\right) e^{-2y} \\ V(y \ll 0) &= \frac{\tilde{n}^2}{4} + \left(\frac{(p+2)\left(q^2 - M^2\right) - pM^2}{4q^2} - \frac{M^2 L^4}{R^2} \frac{p+1-(p-2)q^2/M^2}{(p+1)^2}\right) e^{y}\,. \end{aligned} \tag{A.9}$$

Note that setting $q = 0$ in the expression for $V(y \ll 0)$ is not consistent. This simply reflects the fact that the two limits $y \to -\infty$ and $q \to 0$ do not commute. In any event, comparing these limiting values to those of eq. (3.19) one sees in a $\frac{1}{M}$ expansion that a non-vanishing compact momenta will lead to a larger mass gap than in the $q = 0$ case. Alternatively, one can show that the potential is positive everywhere for $M^2 < 0$, and hence there are no tachyonic excitations. That is the AdS soliton is perturbatively stable against these vector modes with nontrivial τ-dependence.

Finally, we consider the scalar modes for which the ansatz for the polarization must be extended beyond that in eq. (3.23). Again, the transversality constraint (A.1) and consistency of the equations of motion guide us in constructing the following ansatz

$$
\begin{aligned}
H_{\tau\tau}(r) &= -\frac{r^2}{L^2} f(r) H(r) \\
H_{ij}(r) &= \frac{1}{p-1}\frac{r^2}{L^2} H(r)\,\delta_{ij} \qquad i,j = 1,\ldots,p-1 \\
H_{tt}(r) &= \frac{q^2}{M^2} H_{\tau\tau}(r) \\
H_{rr}(r) &= \frac{L^2}{r^2} f^{-1}(r) c(r) \\
H_{rt}(r) &= -iM\, d(r) \\
H_{\tau t}(r) &= -\frac{q}{M} H_{\tau\tau}(r) \\
H_{\tau r}(r) &= iq\, e(r)\,,
\end{aligned}
\tag{A.10}
$$

where $f(r)$ is the function appearing in the metric (2.1), while $c(r)$, $d(r)$ and $e(r)$ are functions fixed by consistency of the equations of motion (3.3). When $q = 0$, this ansatz corresponds to the polarization in eq. (3.23) with $b(r) = 0$. Substituting the ansatz into the linearized Einstein equations (3.3), one can eliminate the functions $c(r), d(r), e(r)$ in terms of $H(r)$ and its derivatives to obtain a single second order equation for $H(r)$. Next one redefines the radial profile by $H(r) = \eta(r)\psi(r)$ with

$$
\eta(r) = \frac{\left(2p\left(1-q^2/M^2\right) r^{p+1} - (p-1)R^{p+1}\right)\sqrt{\frac{r-R}{r\left(r^{p+1}-R^{p+1}\right)}}}{\left(2r^{p+1}\left(p-(p-1)q^2/M^2\right)\left(1-q^2/M^2\right) - (p-1)R^{p+1}\left(1-4q^2/M^2+q^4/M^4\right)\right)}
\tag{A.11}
$$

and changes to the y coordinate as before. In the end, $\psi(r)$ satisfies the Schrödinger equation with the effective potential

$$
\begin{aligned}
V(y) = {}& \frac{1}{4} + \frac{e^{2y}\left(p(p+2)\left(1+e^y\right)^{2(p+1)} - 2p(p+2)\left(1+e^y\right)^{p+1} - 1\right)}{4\left(1+e^y\right)^2\left(\left(1+e^y\right)^{p+1}-1\right)^2} - \\
& - \frac{L^4M^2}{R^2}\frac{\left(\left(1+e^y\right)^{p+1}\left(1+q^2/M^2\right)-1\right)\left(1+e^y\right)^{p-3} e^{2y}}{\left(\left(1+e^y\right)^{p+1}-1\right)^2} - \\
& - \frac{2(p-1)(p+1)^3\left(1+e^y\right)^{p-1} e^{2y}}{\left(\left(1+e^y\right)^{p+1}-1\right)\left(2p\left(1+e^y\right)^{p+1}\left(1+q^2/M^2\right)-p+1\right)^2} - \\
& - \frac{4q^2}{M^2}\frac{e^{2y}\left(p\left(2p-1\right)\left(1+e^y\right)^{2p+2} + p\left(p-1\right)\left(p+1\right)^2\left(1+e^y\right)^{p+1}\right)}{\left(1+e^y\right)^2\left(\left(1+e^y\right)^{p+1}-1\right)\left(2p\left(1+e^y\right)^{p+1}\left(1+q^2/M^2\right)-p+1\right)^2}\,.
\end{aligned}
\tag{A.12}
$$

One can easily see that setting $q = 0$ in this expression reproduces the result in eq. (3.30). This potential behaves in the asymptotic regions as

$$\begin{aligned} V(y \gg 0) &= \frac{(p+1)^2}{4} - \frac{p(p+2)}{2} e^{-y} + \left(\frac{3p(p+2)}{4} - \frac{M^2 L^4}{R^2} (1 + q^2/M^2) \right) e^{-2y} \,, \\ V(y \ll 0) &= \left(\frac{2p(p+2)q^2 + (7p-10)(p+1)M^2}{2pq^2 - (p+1)M^2} + \frac{(p-2)\tilde{n}^2}{4} - \frac{M^2 L^4}{(p+1)R^2} \right) e^{y} \,. \end{aligned} \tag{A.13}$$

Note that in this case, the two limits $y \to -\infty$ and $q \to 0$ do commute for the present calculations. Once again it can be seen that to leading order in $1/M$ the potential near the interior turning point is increased by the inclusion of a non-zero value of the compact momentum however the turning point remains at $y = -\infty$. Further one can show again that there are no tachyonic excitations since the potential is everywhere positive for $M^2 < 0$. Hence as expected, these scalar modes do not yield any instabilities for the AdS soliton when one allows for a nontrivial τ-dependence.

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